# Design of a Low Temperature Differential (LTD) Stirling Engine for Water Transportation

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***Abstract*- Low temperature differential (LTD) Stirling engines (SE) are useful devices when attempting to obtain useful work from low temperature heat sources. In an attempt to implement a renewable small scale LTD SE on a fishing boat, the required energy sources must operate on green energy. This research focuses on the feasibility of a LTD Stirling engine as one of the primary energy sources for a sustainable marine transport. The main focus is on the design and construction of a low temperature differential Stirling engine whose primary source of heating is directed from the sun and cooled through natural convection. This paper reports findings from the LTD SE design analysis.**

***Keywords—Low temperature differential (LTD) Stirling engine, Design, Sustainable Water Transport.***

## I. Introduction

Amid global warming concerns; south pacific island countries are adversely affected by climate change, mainly caused from the by-product of non-renewable energy sources. The south pacific consists of small island nations spread over a large ocean area. Water transportation is largely used for small scale fishing activity and accessing the mainland for replenishment of supplies. Existing engines in small scale fishing boats use petroleum as fuel. Imported fuel add significant expenses to the national economy as well as serious harm to the environment. This research focuses on designing, testing and implementation of low temperature differential (LTD) Stirling engine (SE) on a small scale fishing boat where heat source is directed from the solar energy.

## II. Literature Review

### A. Gamma Engine

The Gamma type Stirling engine uses separated cylinders for the displacer and the power piston as shown in Fig. 1. The gamma configuration allows for large sweep a volume ratio which is the case for all LTD Stirling engines [1]. The sweep volume ratio corresponds to the sweep volume of the displacer relative to that of the power piston [1].The gamma type is relatively easier to make in contrast to the beta type thus making it the suitable choice for LTD Stirling engines with reciprocating displacer.

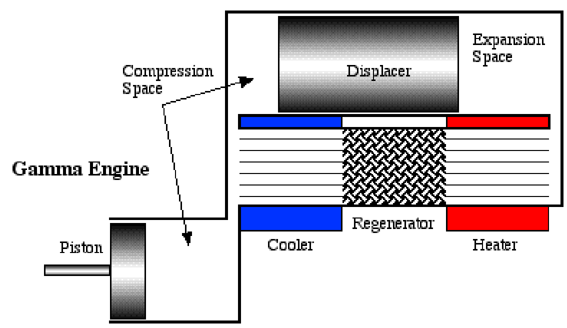


Figure 1. Gamma type Stirling Engine [1].

### B. Power Approximation

Power output can be estimated using different methods that consider factors such as, temperature difference, operation speed, pressure, expansion space volume and compression space volume. A widely used estimate, is the Beale number approximation named after William Beale inventor of the free piston Stirling engine [2]. He showed that the performance of many Stirling engines can be approximated using Eq. 1.

$$P_o = B_n p f V_e \qquad (1)$$

Where $P_o$ is the indicated power in Watts, $p$ is the pressure (bar), $f$ is the frequency (Hz), $V_e$ is the expansion space volume ($cm^3$).

Fig. 2 shows a graph plotted by measuring data from many Stirling engines. The solid line in the middle is typical of many Stirling engines while the upper and lower lines denote unusually high or low performing engines.

### C. Mean Pressure Power Formula

For LTD Stirling engines, the data obtained in Fig. 2 will not provide accurate power estimation calculations. The LTD Stirling engine is a special case and researchers [3, 4] have done detailed work to come up with an improved method of approximating power output. A Beale number correlation was developed to determine the shaft output for LTD Stirling engines more accurately [4] and shown in Eq. 2-3.

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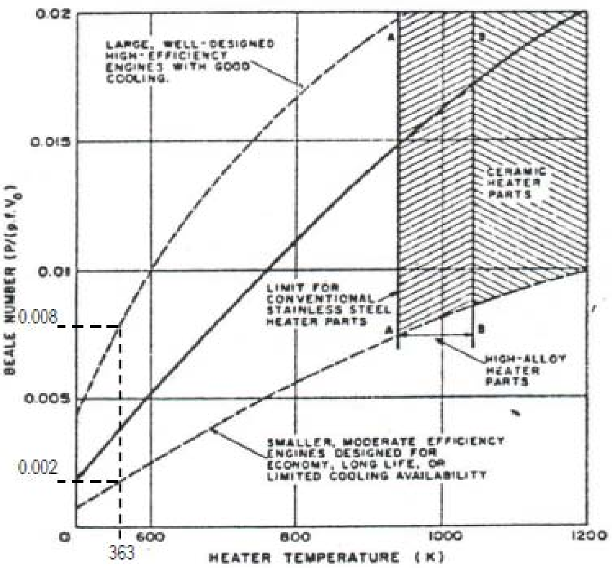



Figure 2. Graph of Beale number vs heater temperature for a range of engines [2].

$$P = p_m V_p f F[\frac{1-\tau}{1+\tau}] \quad (2)$$

$$\tau = \frac{T_c}{T_h} \quad (3)$$

Where $P$ is the engine output power (W); $p_m$ is the mean cycle pressure (N/m$^2$); $f$ is the engine speed or cycle frequency in Hz; $V_p$ is displacement of power piston (m$^3$); $T_c$ is the temperature of cold region; $T_h$ is the temperature of the hot region. A factor $F$ of two (2) may be used for the ideal Stirling cycle, but it does not consider mechanical losses, friction, etc. For practical uses the factor F ranges from 0.25 – 0.35 [4]. A higher value of $F$ should be used for LTD Stirling engines [4].

## III. Stirling Engine Analysis

### A. Schmidt Analysis

The Schmidt theory is an isothermal model for analyzing Stirling engines [5]. It is a simple and effective model during early developments of Stirling engines. It is based on isothermal expansion and compression of an ideal gas. The motion of the displacer and power piston are sinusoidal in nature.

### B. Assumptions of the Schmidt theory

The engine performance can be calculated using a P-V diagram. The volume can be determined using the internal geometry of the engine. When the volume, mass of the working gas and temperature of the working gas is known, the pressure can be calculated using the ideal gas relation shown in Eq. 4.

$$PV = mRT \quad (4)$$

The assumptions of the Schmidt theory are as follows:

- Sinusoidal motion of parts
- Known and constant gas temperatures at all parts of the engine
- No gas leakage
- Working fluid obeys ideal gas law
- At each instant in the cycle, the gas pressure is same throughout the cycle

### C. Engine Indicated Work

Schmidt showed a mathematically exact expression for determining the indicated work per cycle of a Stirling engine [5, 6]. The Schmidt formula can be shown in many forms depending on the notations used. The calculation for a gamma configuration is shown in Eq. 5-7.

$$W = \frac{V_T \bar{p}}{k+1} * (\frac{\pi(1-\tau)k\sin\alpha}{(\sqrt{Y^2 - X^2}) + Y} \quad (5)$$

Where:

$$Y = 1 + \tau + k + \frac{4\tau\chi}{1+\tau} \quad (6)$$

$$X = \sqrt{k^2 - 2k(1-\tau)\cos\alpha + (1-\tau)^2} \quad (7)$$

### D. Instantaneous Pressure and Volume

The changes in volume and pressure inside the displacer cylinder at any given crank angle are governed by Eq. 8-9 [6].

$$V = \frac{V_T}{k+1}(1 + \frac{k}{2}(1 + \cos(\omega t)) + \chi) \quad (8)$$

$$p = \bar{p}(\frac{\sqrt{Y^2 - X^2}}{Y + X\cos(wt - \theta)} \quad (9)$$

*Nomenclature:*

$V_1$ = Displacer Swept Volume

$V_2$ = Piston Swept Volume

$V_d$ = Dead Volume

$T_H$ = Hot Space Temperature

$T_C$ = Cold Space Temperature

$T_D$ = Dead Space Temperature

$\bar{p}$ = Root Mean Cycle Pressure

$p_b$ = External Buffer Pressure

$\omega$ = Angular Velocity of Crankshaft

$\alpha$ = Phase angle

$V_T$ = Total Volume of the Engine

$\tau = {}^{T_c}/_{T_H}$

$k = {}^{V_2}/_{V_1}$

$\chi = {}^{V_D}/_{V_1}$ = Dead Volume Ratio

$\omega_t$ = Instantaneous Angular Position

V = Instantaneous Total Engine Volume

p = Instantaneous Pressure

x = Piston stroke

$x_2$ = Displacer stroke

$D_2$ = Diameter of displacer

L = Length of displacer cylinder

$M_T$ = Total mass of gas

W = Engine Work per cycle

$Ra_D$ = Rayleigh number

g = Acceleration due to gravity

Ts = Temperature of surface of interest

$T_\infty$ = Temperature of surroundings

Pr = Prandtl number

v = Dynamic viscosity

Nu = Nusselt number

$k_2$ = Thermal conductivity

$F_{SA}$ = Surface area of fin

$T_{SA}$ = Total Surface area

$N_F$ = Number of fins

KE = Kinetic energy

I = Inertia

H= Conductive heat transfer coefficient

ω = Angular speed

## IV. Design Calculations

### A. Heat transfer by Natural Convection

Natural convective heat transfer over surfaces depends on the surface geometry and its orientation. It also relies on the variation of the temperature on the surface and the thermo-physical properties of the fluid involved. Natural convective heat transfer over a horizontal cylinder is analyzed using two simple and well defined correlations, Rayleigh number and the Nusselt number [7]. The magnitude of the total natural convective heat transfer between a surface and a fluid is dependent on the flow rate of the fluid. Higher flow rates lead to great heat transfer rate. In natural convection, the fluid flow rate is controlled by the dynamic balance of buoyancy and friction force and not by external means [8]. The buoyancy force is caused by the density difference between the heated (cooled) fluid adjacent to the surface and the fluid surrounding it and is proportional to the density difference. When the warm fluid moves relative to the cold fluid or vice versa, a friction force develops at the contact interface slowing down the flow rate. The Rayleigh number is a product of the Grashof and Prandtl number [7]. The Grashof number is a measure of the relative magnitudes of the buoyancy force and the opposing viscous force acting on the fluid [7]. The average heat transfer coefficient is determined after obtaining the Rayleigh and Nusselt number, considering the following assumptions: steady operation conditions, air is an ideal gas, operating at 1 atmospheric pressure.

Rayleigh number can be obtained using Eq. 10-11.

$$Ra_D = \frac{g\beta(T_s - T_{\infty}) {D_2}^3}{v^2} P_r \quad (10)$$

$$\beta = 2/((T_s - T_\infty) \quad (11)$$

Natural Convection Nusselt Number, Average heat transfer coefficient and Total surface area can be obtained using Eq. 12-14 respectively; which will determine the Rate of heat transfer ($\dot{Q}$).

$$N_u = \left\{ 0.6 + \frac{0.387\left({Ra_D}^{\frac{1}{6}}\right)}{\left[1+\frac{0.559}{P_r}^{\frac{9}{16}}\right]^{\frac{8}{27}}} \right\}^2 \quad (12)$$

$$h = \frac{k2}{D_2}(N_u) \quad (13)$$

$$TSA = \pi D_2 L + (F_{SA})N_F \quad (14)$$

## V. Ross Yoke Mechanism

### A. Design Procedure

The Ross yoke drive is an ingenious mechanism for converting reciprocating motion to rotational motion and is ideal for the gamma type Stirling engine. Estimation of the length of the linkages is dependent on an assumed value of the piston stroke and center to center distance of the displacer to piston cylinder [8]. The mechanism has an advantage of having a reduction of forces on the piston side walls and thus a more efficient choice in contrast to the traditional slider crank mechanism.

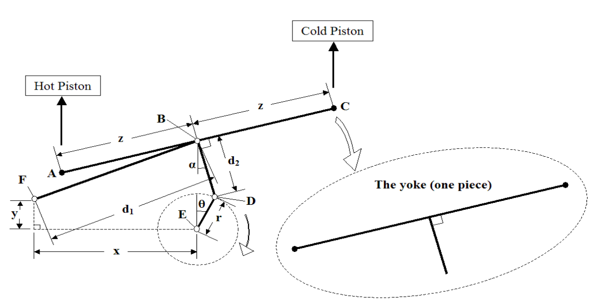



Figure 3. Ross Yoke Drive.

Equation 15 is used for ROSS YOKE mechanism (Fig. 3) calculation where choosing desired Z dimension and stroke length (SL); (Z/14) and (SL/14) become scale factors to calculate other dimensions as shown in Eq. 15-20.

$$d_1^2 = (x + r\sin\theta - d_2\sin\alpha)^2 + (r\cos\theta + d_2\cos\alpha - y)^2 \quad (15)$$

$$x = (Z/14) \times 25 \quad (16)$$

$$y = (Z/14) \times 15 \quad (17)$$

$$r = (SL/4) \times 1.45 \quad (18)$$

$$d_1 = (Z/14) \times 25 \quad (19)$$

$$d_2 = (Z/14) \times 25 \quad (20)$$

*B. Flywheel Analysis*

Flywheels are employed to store energy when the input is more than the requirement and discharge energy when the requirement is more than the input. It is a mechanical form of energy storage and its relative size is governed by the amount of energy required to be stored and its moment of inertia about its rotational axis. As in the design of Stirling engines, flywheels are required to store energy during the power stroke (expansion cycle) which usually is in excess and discharge energy during the return stroke (compression cycle). Kinetic energy of the flywheel is given by Eq. 22 [9]:

$$KE = 0.5I\omega^2 = 0.5mk^2\omega^2 \quad (21)$$

As the speed of the flywheel changes from $\omega_1$ to $\omega_2$, the fluctuation in energy is given by:

$$\Delta E = \text{Maximum K.E} - \text{Minimum K.E} = 0.5 \times I \times (\omega 1)^2 - 0.5 \times I \times (\omega 2)^2 = 0.5 \times I \times (\omega 1 + \omega 2)(\omega 1 - \omega 2) = I\omega(\omega 1 - \omega 2) = I\omega^2((\omega 1 - \omega 2)/\omega) = mk^2\omega^{(2C_s)} = 2EC_s \quad (22)$$

Substituting k = r, the mass of the flywheel can be determined.

*C. Static Analysis*

Due to the gas (i.e. air) load on the piston, a torque is applied on the output shaft through the linkage mechanism as shown in Fig. 4. At different pressures, the gas load varies and thus torque on the shaft can be computed for different crank angle positions.

By solving for the respective forces the torque on the output shaft can be determined.

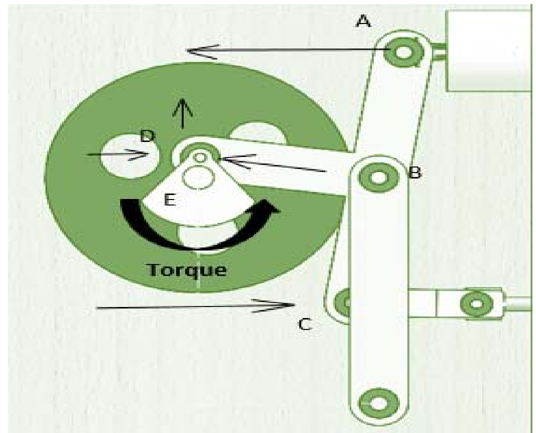



Figure 4. Ross Linkage Static Analysis.

## VI. RESULTS

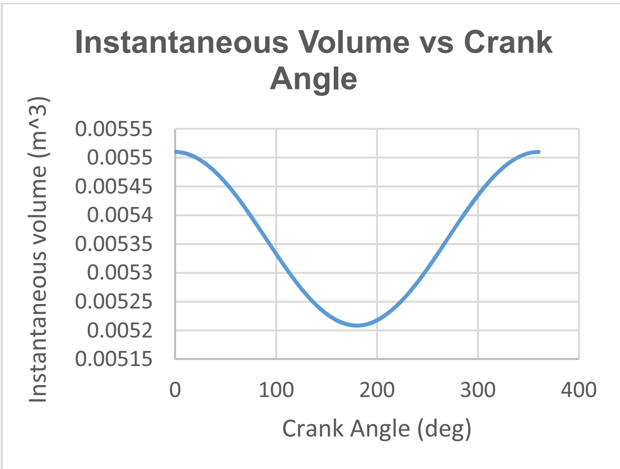
Table I is a summary of the preliminary design calculations along with Schmidt analysis of the Net Work per cycle, which also includes the heat transfer calculations for the heat sink, design calculations of the Ross yoke linkage and flywheel sizing.

TABLE I: SUMMARIZED DESIGN CALCULATIONS

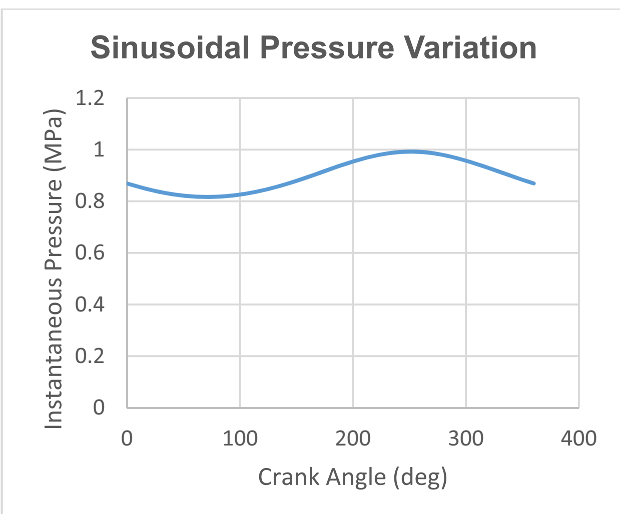
| | | | |
|---|---|---|---|
| ***Engine Sizing*** | | | |
| $T_c$ (K) | 303 | $V_p$ ($m^3$) | 0.00030159 |
| $T_h$ (K) | 373 | $V_e$ ($m^3$) | 0.0047 |
| $\tau$ | 0.81 | x(m) | 0.0059 |
| $B_n$ | 0.0036 | $x_2$ (m) | 0.0056 |
| P (Watts) | 350 | $D_2$ (m) | 0.326 |
| Pressure (MPa) | 0.9 | L (m) | 0.17 |
| Frequency (Hz) | 8 | | |
| ***Schmidt Analysis*** | | | |
| $M_T$ (kg) | 0.058 | W (J) | 43.60 |
| ***Heat Transfer*** | | | |
| $T_f$ (°C) | 55 | $N_u$ | 72.13 |
| Pr | 0.7228 | h (W/m.°C) | 5.73 |
| μ (kg/m.s) | 1.963x10^-5 | β | 1/328 |
| $k_2$ (W/m.°C) | 0.027 | $Ra_D$ | 218.17x10^6 |
| ν (m^2/s) | 1.798x10^-5 | $T_{SA}$ (m^2) | 2.97 |
| | | $\dot{Q}$(KW) | 1.069 |
| ***Ross Yoke mechanism*** | | | |
| z (mm) | 137.5 | $d_1$ (mm) | 245.54 |
| $d_2$ (mm) | 147.32 | | |
| ***Flywheel*** | | | |
| D (mm) | 250 | Material | Cast Iron |
| Thickness (mm) | 15 | | |

*A. Sinusoidal Pressure and Volume Variation*

Using equations 8 and 9 one can determine the pressure variations in the engine. These variations are dependent on the crank angle and are sinusoidal in nature. Using Fig. 5, one can determine the maximum and minimum volume and pressure in the engine.




(a)




(b)

Figure 5. Graph of (a) Instantaneous engine volume and (b) Instantaneous engine pressure.

*B. Simulations results*

Using Solidworks software, the following simulations were carried out:

- Displacer cylinder stress analysis
- Pressure Analysis
- Heat sink simulation
- Dynamic Simulations

*C. Displacer Cylinder Stress Analysis*

Fig. 6 shows a simulation result of stress analysis on the displacer cylinder. The high pressure gas contained in the vessel induces high axial and radial forces on the cylinder walls. The thickness of the walls should be sufficient to prevent failure. With 1MPa maximum working pressure, the maximum Von-Mises stress is 149MPa. Stainless steel has a yield stress of around 172MPa which is greater than the maximum stress induced by the gas pressure.

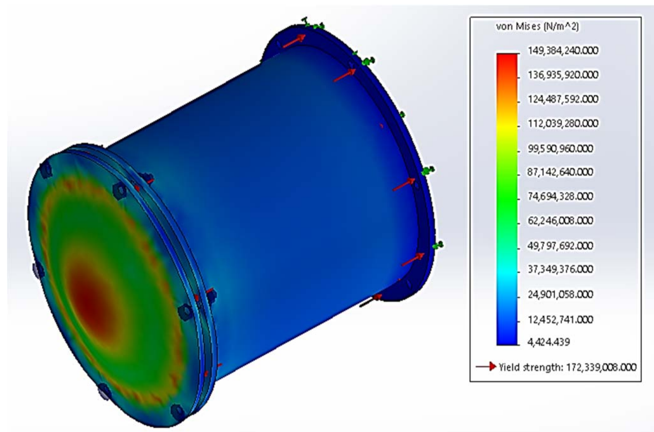



Figure 6. Displacer cylinder stress analysis.

*D. Pressure Analysis*

Using Schmidt theory, the maximum pressure in the vessel is 1MPa as shown in Fig. 5 (b). Using Solidworks flow simulation, a pressure analysis of the fluid was carried out. The fluid was modelled in a closed domain with an initial pressure of 0.9MPa and temperature of 298K. Heat is applied to the flat face of the cylinder which in turn raises the temperature of the enclosed fluid. The simulation is based on natural convection within a closed domain with gravity defined along the axis of the cylinder. Pressure analysis shown in Fig. 7 indicates that the maximum attained pressure is 1MPa.

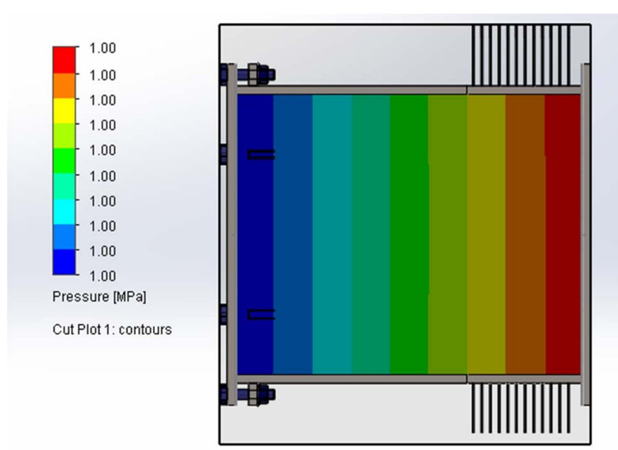



Figure 7. Maximum pressure simulation.

## E. Heat Sink Simulation

Heat transfer is the driving force for the required pressure difference to operate the engine. A heat sink is needed to achieve the desired cooling. Engine cooling is achieved by means of convective heat transfer on the outside surface of the cylinder. To increase rate of heat transfer, the one of the option is to increase the surface area. Using Solidworks simulation, a thermal study is setup to evaluate the performance characteristics of the heat sink.

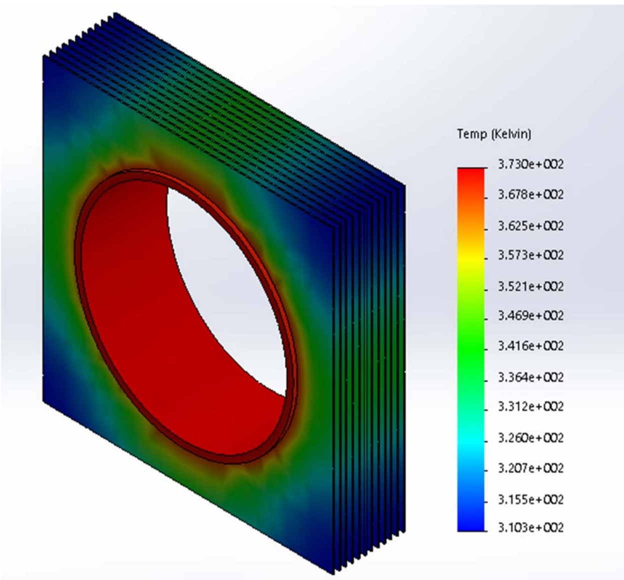



Figure 8. Heat sink simulation.

The thermal study is a transient analysis, making the solution time dependent. Applying a temperature of 373K on the inner cylindrical face at the cold end of the displacer cylinder, and a heat transfer coefficient of 5.77W/m^2K on the all external surfaces, the temperature distribution is obtained. The value for the convective heat transfer coefficient through natural convection is derived from hand calculations using natural convective heat transfer model on a horizontal cylinder. The thermal analysis ran for 300 seconds. From Fig. 8, it is evident that the desired cooling has not been achieved after 5 minutes. However to solve this a regenerator was used in this design.

## F. Dynamic Simulations

*Engine speed*

Using static analysis, the torque on the shaft (due to the gas load on the piston) was computed. The maximum speed of the engine is expected to be when the crank has turned 90 degrees. The angular acceleration was calculated; based on the torque value, and the moment of inertia of crankshaft assembly. The crankshaft would accelerate from zero to maximum speed when the crank has reached maximum speed position. Dynamic motion loads can be easily analyzed using Solidworks motion analysis tool. Through motion analysis, the crankshaft assembly was accelerated from zero to maximum speed within 90 degrees. A linear interpolation of angular velocity with respect to time was defined which governs the motion of the crankshaft as shown in Fig. 9 (i.e. the engine speed as a function of time). The engine accelerates from zero to about 650 deg/sec within 0.1 seconds. The angular displacement is equivalent to 90 degrees turn of the crank arm.

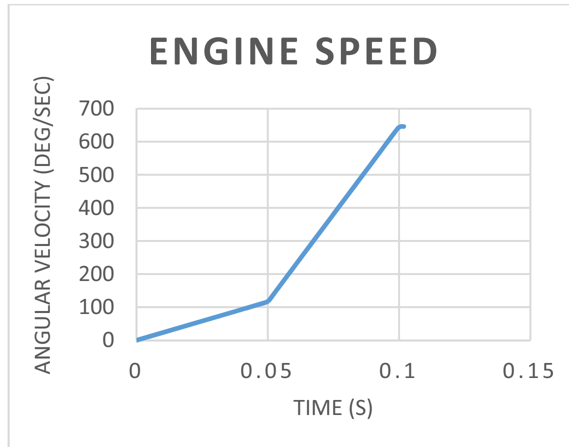



Figure 9. Graph of Angular speed vs time.

*Engine torque*

Motion analysis can also plot a torque diagram based on dynamic analysis. A torque plot was generated; based on the speed of crankshaft at different points in time. Constant torque is observed during periods of acceleration. Zero torque is observed during periods of constant speed. Fig. 10 shows a torque plot as function of time which is a result of the angular velocity data defined in Solidworks motion analysis.

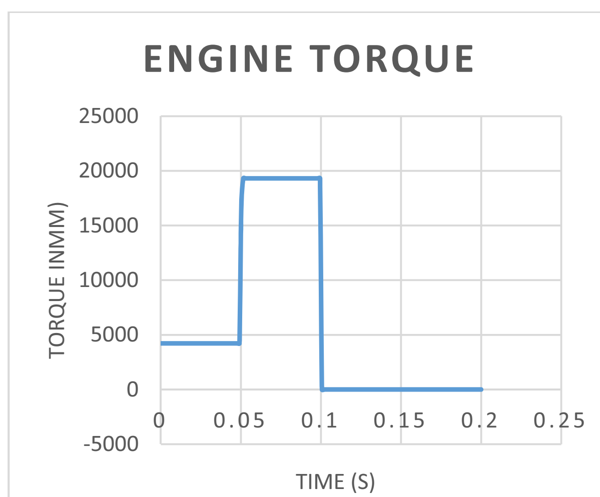



Figure 10. Torque diagram.

*Power Output*

Motion analysis can also plot the power consumption of a rotating body. In this case, the expansion power is plotted as function of time as shown in Fig. 11. Peak expansion of power occurs at 90 degrees turn of the crank arm. Peak expansion of power is 215 Watts resulting from the expansion of the gas (i.e. air) inside the displacer cylinder. Overall assembly design of the SE is shown in Fig. 12.

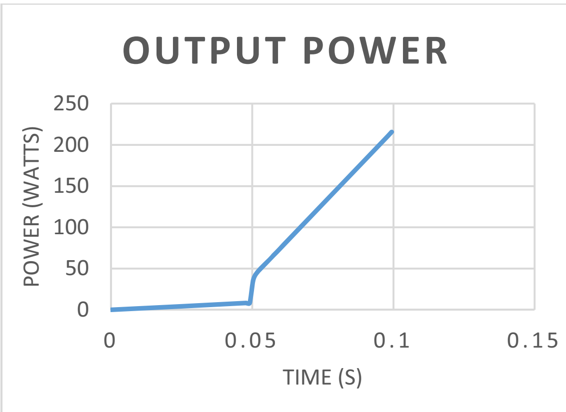



Figure 11. Graph of expansion power.

Figure 12. Assembly design of the proposed SE.

## VII. Conclusion

Economic analysis provides a suitable method to determine feasibility of LTD Stirling engines for water transportation. In summary, LTD Stirling engines require large sweep volume ratio's which in turn determines the overall engine size. In most cases, LTD Stirling engines are quite large. The engine weights 117kg with an expansion power of 215 Watts. The power to weight ratio is considerably low when compared to an outboard motor of similar power output. Apparently; LTD Stirling engines are not feasible for water transportation. Reducing the weight of the engine, increasing the operating temperature difference and lighter materials (i.e. composite fiber materials) to withstand the working conditions should be considered to improve the power to weight ratio. However, LTD Stirling engines are still could be a best choice while considering green and renewable energy sources for the sustainable marine transport.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