A perfect Mathematical Formulation of Fuzzy Transportation Problem Provides an Optimal Solution Speedily

K. Ramanuja Rao[1], Rajnesh K Mudaliar[2]
[1]Assistant Professor of Mathematics,
[2]Lecturer of Mathematics
Department of Mathematics,
Fiji National University, Lautoka, Fiji,
[1]Ramanuja.kotti@fnu.ac.fj, [2]Rajnesh.Mudaliar@fnu.ac.fj

M. K.Hasan[3], M. K. Alam[4]
Assistant Professor of Mathematics
Department of Mathematics and Statistics
Bangladesh University of Business and Technology
[3]krul.habi@yahoo.com, [4]khairul638@yahoo.com

Abstract— The basic transportation problem was originally developed by Frank Lauren Hitchcock [11], [16]. The transportation problem which transports goods from m sources to n different destinations to minimize total shifting cost. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. In a fuzzy transportation problem, all parameters are fuzzy number. The aim of Fuzzy transportation is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and capacity and cost of edges are represented as fuzzy numbers. Fuzzy numbers may be normal or subnormal, Triangular or Trapezoidal or any Fuzzy Russell fuzzy number. Some fuzzy numbers are not directly comparable. In real life case, transportation unit costs have not been precisely determined beforehand, but they are specified by the fuzzy parameters. This paper presents a simple and efficient method that is better than the existing methods, easy to understand and also can give an optimal solution. The proposed method, Improved Zero Point Method (IZPM) [10], is used for solving unbalanced fuzzy transportation problems by assuming that a decision maker is uncertain about the precise values of the transportation cost only. In this paper I would like to present, how a fuzzy transportation problem can formulate perfectly for reaching to the optimal solution.

Keywords— Fuzzy Sets, Fuzzy Numbers, Arithmetic Operation on Fuzzy Number, Fuzzy Transportation Problem

1.0 Introduction

The idea of fuzzy set was introduced by Zadeh [29] in 1965. The linear programming formulation and the associated systematic method for solution were first given by Dantzin [9] then after 1960's Bellman and Zadeh [4] proposed the concept of decision making in fuzzy environment. Lai and Hwang [19] others consider the situation where all parameters are fuzzy. There are several solution methods for transportation problem when prices and quantities are given as crisp numbers [13], [17]. Several variations of transportation methods have been using with the table methods such as the northwest corner method, the shortcut methods and Russell's approximation method [13], [17]. Some others [1], [5], [27] have been using special techniques for linear programming problem because the classic single objective transportation problem is a special case of the linear programming problem. However, recently fuzzy programming approach started to use the optimal solutions of multi objective or single objective transportation problem [6], [18], [19], [26], [27]. For instance, Wahed [26] represent the fuzzy programming approach to determine the compromise solution of multi objective transportation problem. Kikuchi [18] proposed a simple adjustment method that finds the most appropriate set of crisp numbers. Wahed [27] presented an interactive fuzzy goal programming approach to determine the preferred compromise solution for multi objective transportation problems. In reality, it is not possible to determine both quantities and transportation unit prices, but the fuzzy numbers gives best approximation of them. A model solving the transportation problem is given in [19] when quantities are fuzzy and prices are crisp. Again in [18] is given a methods determining quantities that is satisfied the higher satisfactory level while quantities is only fuzzy. OhEigeartaigh [17], [22] considered the case where the membership functions of the fuzzy demands are triangular forms for transportation problems and solved it using table method. Geetha and Nair [14] formulated a stochastic version of the time minimizing transportation problem and developed an algorithm based on parametric programming to solve it when transportation time is considered to be independent, positive normal random variables. Chanas, Kolodziejczyk and Machaj [6] are analyzed the transportation problem with fuzzy supply values of deliverers and with fuzzy demand values of the receivers. Liu and Kao [20] developed a procedure to derive the fuzzy objective value of the fuzzy transportation problem, in that the cost coefficients and the supply

978-1-5090-5753-5/16 $31.00 © 2016 IEEE
DOI 10.1109/APWC.on.CSE.2016.49

and demand quantities are fuzzy numbers basing on extension principle. Ahlatioglu, Sivri and Güzel [2] proposed a solution algorithm finding all fuzzy optimal solution of the transportation problem that the cost coefficients and the supply and demand quantities are fuzzy numbers. Chanas and Kuchta [7] proposed an algorithm that solves the transportation problem with fuzzy supply and demand values and integrality condition imposed on the solution. Gen [15] describes an implementation of genetic algorithm to solve Bicriteria Solid Transportation Problem.

In this paper I would like to discuss thoroughly regarding the formulation of a fuzzy transportation problem. A numerical example has been solved for showing how can we reach to the optimal solution. Theory of fuzzy transportation problem, Fuzzy Russell's method, Fuzzy Vogel's Approximation Algorithm (FVAM) and Fuzzy Zero Point Method (FZPM) are discussed theory of fuzzy transportation problem theory of fuzzy transportation problem theory of fuzzy transportation problem in section 2. In section 3 In this section we are presenting an algorithm to find the fuzzy feasible solution and fuzzy optimal solution of a fuzzy transportation problem and section 4 we have discussed results and conclusion respectively.

2.0 Preliminaries

L. A. Zadeh [29] advanced the fuzzy theory in 1965; the theory proposes mathematical techniques for dealing with the concepts and problems that have much possible solution. In year 1974 concept of mathematical programming on a general level was first proposed by Tanaka et al. in the frame work of fuzzy decision. In this section we shall introduce ranking method for trapezoidal fuzzy numbers that will be needed. Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. It seems that parametric methods of comparing fuzzy numbers, especially in fuzzy decision making theory, are more efficient that non parametric methods. In Cheng [8] centroid point method, fuzzy numbers are compared according to their Euclidean distance for the origin. Negative fuzzy number in Cheng's centroid point method where not compared. Abbasbandy and Asad [3] found the chu and Tsao's area method occasionally causes non intuitive ranking. They presented the sign distance method. But their method was non parametric methods for comparing fuzzy numbers have some drawbacks in practice.

2.1 In Fuzzy Russell's method we use Yager's [28] Ranking method. According to Yager ranking function:

A ranking function $\mathfrak{R}: F(\mathbb{R}) \to \mathbb{R}$, where $F(\mathbb{R})$ is set of all fuzzy numbers defined on set of real numbers, which maps each fuzzy number in to a real number. Let $A = \left[[a_1, a_2, a_3, a_4]\right]$ be any TFN then

$$\mathfrak{R}(A) = \frac{w_1(a_1 + b_1 + c_1 + d_1)}{4}$$

- ***i.*** If $\mathfrak{R}(A) > R(B)$ then we say $A \succ B$
- ***ii.*** If $\mathfrak{R}(A) < R(B)$ then we say $A \prec B$
- ***iii.*** If $\mathfrak{R}(A) = \mathfrak{R}(B)$ then we say $A \approx B$ [then comparison is not possible]

2.2 It is clear that non parametric methods for comparing fuzzy number have some drawbacks in practices [12]. According to above mentioned definition of fuzzy number:

$Aw = [\inf \mu(r), \sup \mu(r)]$, where $0 \le r \le w$ also w is arbitrary constant with $0 \le w \le 1$. Then the value of $wA^{\alpha} = [\inf \mu(\alpha), \sup \mu(\alpha)], 0 \le \alpha \le w$ is assigned for a decision level higher than "α" which is calculated as follows:

$$M^{\alpha}(A) = \frac{1}{2}\int_{\alpha}^{w} \{\inf A(r) + \sup A(r)\}\, dr; \quad \text{where } 0 \le \alpha \le 1 \qquad (1)$$

This quantity will be used as a basis for comparing fuzzy numbers in decision level higher than α.

2.3 Measure of Fuzzy Numbers

A measure of a fuzzy number A is a function$M: \mathcal{F} \to \mathbb{R}+$, where $\mathcal{F}$ denotes the set of all fuzzy numbers. For each fuzzy number$A$, this function assigns a non-negative real number $M(A)$ that expresses the measure of A. This means that a fuzzy number is obtained by the average of two side areas (left and right side area), from membership function to α axis.

2.3.1 $M(A) = A$ if and only if A is a crisp number

2.3.2 $A \le B$ if and only if $M(A) \le M(B)$**,** $M^{\alpha}(wA) = 0$ if and only if $\alpha \ge w$, Then we get,

$$M^{\alpha}(A) = \frac{1}{2}\int_{\alpha}^{w} \{\inf A(r) + \sup A(r)\}\, dr$$

$$= \frac{1}{2}\int_{\alpha}^{w} \inf A(r)\, dr + \frac{1}{2}\int_{\alpha}^{w} \sup A(r)\, dr \qquad (2)$$

2.4 Remarks: If Aw_1 and Bw_2 are two arbitrary fuzzy numbers with $w_1, w_1 \in [0, 1]$ and $\alpha \in [0, 1]$, then

2.4.1 $Aw_1 \ge Bw_2$ if and only if $M^{\alpha}(A) \ge M^{\alpha}(B)$

2.4.2 $Aw_1 \le Bw_2$ if and only if $M^{\alpha}(A) \le M^{\alpha}(B)$

2.4.3 $Aw_1 = Bw_2$ if and only if $M^{\alpha}(A) = M^{\alpha}(B)$ [then comparison is not possible]

If α is close to one, the pertaining decision is called a "*high level decision*", in which case only parts of the two fuzzy numbers, with membership values between α and 1, will be compared. On the other hand if α is close to zero, the pertaining decision is referred to as a "*low level decision*" since members with membership values lower than both the fuzzy numbers are involved in the comparison.

2.5 Fuzzy Transportation Problem

Consider a fuzzy transportation with m sources and n destinations with fuzzy numbers. Let $a_i = \left[\left[a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, a_i^{(4)}\right]\right]$ $(a_i \geq 0)$ be the fuzzy availability at source i and $b_j = \left[\left[b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, b_j^{(4)}\right]\right]$ $\left(b_j \geq 0\right)$ be the fuzzy requirement at destination j. Let $c_{ij} = \left[\left[c_{ij}^{(1)}, c_{ij}^{(2)}, c_{ij}^{(3)}, c_{ij}^{(4)}\right]\right]$ be the fuzzy unit transportation cost from source i to destination j. Let $x_{ij} = \left[\left[x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}, x_{ij}^{(4)}\right]\right]$ be the number of fuzzy units (Decision variables) to be transported from source i to destination j. Then The problem is to determine a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimize.

The mathematical formulation of the fuzzy transportation problem whose parameters are fuzzy numbers under the case that the total supply is equivalent to the total demand is given by

$$Z_{min} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraint

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \dots \dots \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \dots \dots \dots, n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

and $x_{ij} \geq 0$ for all i and j

This above fuzzy transportation problem can be represented as follows:

Destinations

Sources	1	2	……	n	Supply
1	c_{11} x_{11}	c_{12} x_{12}	……	c_{1n} x_{1n}	a_1
2	c_{21} x_{21}	c_{22} x_{22}	……	c_{2n} x_{2n}	a_2
⋮	⋮	⋮	⋮	⋮	⋮
m	c_{m1} x_{m1}	c_{m2} x_{m2}	……	c_{mn} x_{mn}	a_m
Demand	b_1	b_2	……	b_n	

2.6 Definition: [24] Any set of fuzzy non negative allocations x_{ij} which satisfies (in the sense equivalent) the row and the restrictions is known as *fuzzy feasible solution*.

2.7 Definition: [24] A fuzzy feasible solution to a fuzzy transportation problem with m sources and n destinations is said to be *fuzzy basic feasible solution* if the number of positive allocations are $(m + n - 1)$.

2.8 Definition: [24] A fuzzy feasible solution to a fuzzy transportation problem with m sources and n destinations is said to be *fuzzy degenerate basic feasible solution* if the number of positive allocations is less than$(m + n - 1)$.

2.9 Definition: [24] A fuzzy feasible solution to a fuzzy transportation problem with m sources and n destinations is said to be *fuzzy non degenerate basic feasible solution* if it contains exactly $(m + n - 1)$ occupied cells.

2.10 Definition: [24] A fuzzy feasible solution is said to be fuzzy optimal solution if it minimize the total fuzzy transportation cost.

2.11 Theorem: [21] Existence of a fuzzy feasible solution.

Statement: The necessary and sufficient condition for the existence of fuzzy feasible solution to the fuzzy transportation is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Proof: Necessary condition: Let there exist a fuzzy feasible solution to the fuzzy transportation problem

$$Z_{min} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constrain

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \dots \dots \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \dots \dots \dots, n$$

$$x_{ij} \geq 0 \quad \text{for all } i, j.$$

So we get

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \dots \dots \dots, m$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i$$

Also from

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \dots\dots\dots, n$$

We have

$$\sum_{j=1}^{n}\sum_{i=1}^{n} x_{ij} = \sum_{j=1}^{n} b_j$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \dots\dots\dots, n$$

Hence

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

<u>Sufficient condition</u>**:** Let us consider that $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. We have to distribute the supply at the i-th source in proportion to the requirements of all destinations. Let $x_{ij} = k_i b_j$, where k_i is the proportionality factor for the i-th source. Since the supply must be completely distributed.

$$\sum_{i=1}^{m} x_{ij} = k_i \sum_{j=1}^{n} b_j$$

Therefore,

$$x_{ij} = k_i b_j = \frac{a_i}{\sum_{j=1}^{n} b_j}\, b_j$$

$$\sum_{j=1}^{n} x_{ij} = k_i \sum_{j=1}^{n} b_j = \frac{a_i}{\sum_{j=1}^{n} b_j} \sum_{j=1}^{n} b_j = a_i$$

$$\sum_{i=1}^{m} x_{ij} = \frac{b_j}{\sum_{i=1}^{m} a_i} \sum_{i=1}^{m} a_i = b_j$$

which shows that all the constrains are satisfied. Since a_i and b_j are positive, x_{ij} determined must be positive. Therefore the fuzzy transportation problem yields a fuzzy feasible solution.

2.12 Theorem: [25] The dimension of the basis of a fuzzy transportation problem are $(m+n-1)$. That a fuzzy transportation has only $(m+n-1)$ independent structural constraints and its basic feasible solution has only $(m+n-1)$ positive components.

Proof: Consider a fuzzy transportation problem with m sources and n destinations,

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constrain

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \dots\dots\dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \dots\dots\dots, n$$

and $x_{ij} \geq 0$ for all i and j.

Let us assume that a fuzzy transportation has m rows (supply constraint equations) and n columns (demand constraint equations). Therefore there are totally $(m+n)$ constraint equations.

This is due to the condition that $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ which is the least requirement constraint. Therefore one of $(m+n)$ constraints can always be derived from the remaining $(m+n-1)$. Thus there exists only $(m+n-1)$ independent constraints and its basic feasible solution has only $(m+n-1)$ positive components.

2.13 Fuzzy Vogels Approximation Algorithm (FVAM)

Setp 1: From the fuzzy transportation table determine the penalty for each row and column. The penalties are calculated for each row column by substituting the lowest cost element in that row or column for the next cost element in the same row or column. Write down the penalties below and aside of the rows and columns respectively of table.

Step 2: Identify the column or row with largest fuzzy penalty. In case of tie, break the tie arbitrary. Select a cell with minimum fuzzy cost in the selected column (or row) and assign the maximum units possible by considering the demand and supply position corresponding to the selected cell.

Step 3: Delete the column (or row) for which the supply and demand requirements are met.

Step 4: Continue steps 1 to 3 for the resulting fuzzy transportation table until the supply and demand of all sources and destinations have been met.

2.14 Fuzzy Zero Point Method (FZPM)

Setp 1: Construct the fuzzy transportation table for the given fuzzy transportation problem and then, convert it into a balanced one (if it is not).

Step 2: Subtract each row entries of the fuzzy transportation table from the row minimum.

Step 3: Subtract each column entries of the resulting fuzzy transportation table after using the Step 2. From the column minimum.

Step 4. Check if each column fuzzy demand is less than to the sum of the fuzzy supplies whose reduced costs in that column are fuzzy zero. Also, check if each row fuzzy supply is less than to sum of the column fuzzy demands whose reduced costs in that row are fuzzy zero. If so, go to Step 7. (Such reduced table is called the allotment table). If not, go to Step 5.

Step 5. Draw the minimum number of horizontal lines and vertical lines to cover all the fuzzy zeros of the reduced fuzzy transportation table such that some entries of row(s) or / and column(s) which do not satisfy the condition of the Step4. are not covered.

Step 6: Develop the new revised reduced fuzzy transportation table as follows: (i) Find the smallest entry of the reduced fuzzy cost matrix not covered by any lines. (ii) Subtract this entry from all the uncovered entries and add the same to all entries lying at the intersection of any two lines. and then, go to Step 4.

Step 7. Select a cell in the reduced fuzzy transportation table whose reduced cost is the maximum cost. Say (α ,β) . If there are more than one, then select anyone.

Step 8. Select a cell in the α -row or/ and β − column of the reduced fuzzy transportation table which is the only cell whose reduced cost is fuzzy zero and then, allot the maximum possible to that cell. If such cell does not occur for the maximum value, find the next maximum so that such a cell occurs. If such cell does not occur for any value, we select any cell in the reduced fuzzy transportation table whose reduced cost is fuzzy zero.

Step 9. Reform the reduced fuzzy transportation table after deleting the fully used fuzzy supply points and the fully received fuzzy demand points and also, modify it to include the not fully used fuzzy supply points and the not fully received fuzzy demand points.

Step 10. Repeat Step 7 to Step 9 until all fuzzy supply points are fully used and all fuzzy demand points are fully received.

Step 11. This allotment yields a fuzzy solution to the given fuzzy transportation problem.

3 Formulation

In this section we are presenting an algorithm to find the fuzzy feasible solution and fuzzy optimal solution of a fuzzy transportation problem.

Consider a fuzzy transportation problem which has described in this paper. For this fuzzy transportation problem a_i and b_j are fuzzy quantities which are represented by triangular fuzzy numbers $a_i = \left[\left[-\infty, a_i^{(2)}, a_i^{(3)}\right]\right]$ and $b_j = \left[\left[\, b_j^{(1)}, b_j^{(2)}, \infty\right]\right]$. Also the transportation cost per unit is given by $c_{ij} = \left[\left[-\infty, c_{ij}^{(2)}, c_{ij}^{(3)}\right]\right]$.

Then the membership function of a_i, b_j and c_{ij} are as follows:

$$\mu(a_i) = \begin{cases} 1 & ; \quad a_i \le a_i^{(2)} \\ \dfrac{a_i - a_i^{(3)}}{a_i^{(2)} - a_i^{(3)}} & ; \quad a_i^{(2)} \le a_i \le a_i^{(3)} \\ 0 & ; \quad a_i^{(3)} \le a_i \end{cases}$$

$$\mu(b_j) = \begin{cases} 0 & ; \quad b_j \le b_i^{(1)} \\ \dfrac{b_j - b_j^{(1)}}{b_j^{(2)} - b_j^{(1)}} & ; \quad b_j^{(1)} \le b_j \le b_j^{(2)} \\ 1 & ; \quad b_j^{(2)} \le b_j \end{cases}$$

$$\mu(c_{ij}) = \begin{cases} 1 & ; \quad c_{ij} \le c_{ij}^{(2)} \\ \dfrac{c_{ij} - c_{ij}^{(3)}}{c_{ij}^{(2)} - c_{ij}^{(3)}} & ; \quad c_{ij}^{(2)} \le c_{ij} \le c_{ij}^{(3)} \\ 0 & ; \quad c_{ij}^{(3)} \le c_{ij} \end{cases}$$

There are one $a_i = \mu_i^{-1}(\alpha)$ and $b_j = \mu_j^{-1}(\alpha)$ each of them exists in $0 < \alpha < 1$ and the membership function are monotone for$0 < \alpha < 1$. If these numbers are explained with $\alpha - \text{parameter}$, then we obtain from $\mu(a_i) = \alpha$ and $\mu(b_j) = \alpha$:

$$a_i = \mu_i^{-1}(\alpha) = a_i^{(3)} - \alpha\left(a_i^{(3)} - a_i^{(2)}\right)$$

$$b_j = \mu_j^{-1}(\alpha) = b_j^{(1)} + \alpha\left(b_j^{(2)} - b_j^{(1)}\right)$$

For increasing of $\alpha - \text{parameter}$ is caused that the quantity of a_i in i[th] source decreases and the quantity of b_j in j[th] destination increases.

The total quantities in the sources is

$$\sum_{i=1}^{m} a_i = \sum_{i=1}^{m} \left(a_i^{(3)} - \alpha\left(a_i^{(3)} - a_i^{(2)}\right)\right) \tag{5}$$

The total demand in the destination is

$$\sum_{j=1}^{n} b_j = \sum_{i=1}^{m} \left(b_j^{(1)} + \alpha\left(b_j^{(2)} - b_j^{(1)}\right)\right) \tag{6}$$

The membership functions of these numbers are shown in the following figure:

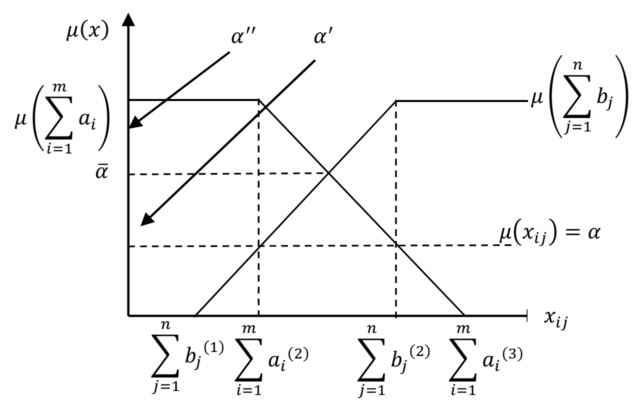

Figure: The membership function of supply and demand

The total demand and total supply satisfy $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, then

$$\bar{\alpha} = \frac{\sum_{i=1}^{m} a_i^{(3)} - \sum_{j=1}^{n} b_j^{(1)}}{\sum_{i=1}^{m}\left(a_i^{(3)} - a_i^{(2)}\right) + \sum_{j=1}^{n}\left(b_j^{(2)} - b_j^{(1)}\right)} \tag{7}$$

Where $\sum_{i=1}^{m}\left(a_i^{(3)} - a_i^{(2)}\right) + \sum_{j=1}^{n}\left(b_j^{(2)} - b_j^{(1)}\right) > 0$

α must satisfy the equation $\alpha = \max\min\left\{\mu\left(\sum_{i=1}^{m} a_i\right), \mu\left(\sum_{j=1}^{n} b_j\right)\right\}$,

$\bar{\alpha} \in [0,1]$.

If $\bar{\alpha} \le 0$ then the maximum satisfactory level becomes balanced by constructing the artificial source.

Again, if $0 < \bar{\alpha} \le 1$ then the problem becomes balanced by constructing the artificial destination. Using linear transformation, the corresponding satisfactory level may be found as

$$\alpha'' = \frac{\sum_{i=1}^{m} a_i^{(3)} - \sum_{j=1}^{n} b_j^{(1)} - \sum_{j=1}^{n}\left(b_j^{(2)} - b_j^{(1)}\right)\alpha'}{\sum_{i=1}^{m}\left(a_i^{(3)} - a_i^{(2)}\right)} \quad \text{where } \alpha'' \le \bar{\alpha} \le \alpha' \tag{8}$$

For $0 \le \beta \le \alpha$ if $\sum_{j=1}^{n} b_j^{(2)} < \sum_{i=1}^{m} a_i^{(3)}$, then

$$\beta = \frac{\sum_{j=1}^{n} b_j^{(2)} - \sum_{i=1}^{m} a_i^{(3)}}{\sum_{i=1}^{m}\left(a_i^{(2)} - a_i^{(3)}\right)}$$ and if $\sum_{j=1}^{n} b_j^{(2)} \ge \sum_{i=1}^{m} a_i^{(3)}$, then $\beta = 0$.

If $\bar{\alpha} \ge 1$ then by constructing the demand center to

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = \sum_{i=1}^{m}\left(a_i^{(3)} - \left(a_i^{(3)} - a_i^{(2)}\right)\alpha\right) - \sum_{j=1}^{n}\left(b_j^{(1)} - \left(b_j^{(2)} - b_j^{(1)}\right)\alpha\right)$$

quantity, the problem can be balanced.

Then the satisfactory level (α) can be increases as much as $(\alpha = 1)$ but in solution involving quantities out of intervals the (α) value because $(\alpha < 1)$.

Thus, to become balanced the problem, $\bar{\alpha}$ is defined to the sum of the quantities in sources $\sum_{i=1}^{m}\left(a_i^{(3)} - \left(a_i^{(3)} - a_i^{(2)}\right)\alpha\right)$ and to the sum of demand quantities $\sum_{j=1}^{n}\left(b_j^{(1)} + \left(b_j^{(2)} - b_j^{(1)}\right)\alpha\right)$. Then the artificial (or dummy) source or demand centers are established with respect to the structure of the problem. Therefore the problem would become balanced.

Then the satisfactory level (α) can be increases as much as $(\alpha = 1)$ but in solution involving quantities out of intervals the (α) value because $(\alpha < 1)$.

Thus, to become balanced the problem, $\bar{\alpha}$ is defined to the sum of the quantities in sources $\sum_{i=1}^{m}\left(a_i^{(3)} - \left(a_i^{(3)} - a_i^{(2)}\right)\alpha\right)$ and to the sum of demand quantities $\sum_{j=1}^{n}\left(b_j^{(1)} + \left(b_j^{(2)} - b_j^{(1)}\right)\alpha\right)$. Then the artificial (or dummy) source or demand centers are established with respect to the structure of the problem. Therefore the problem would become balanced.

3.1 An Illustrative Example

Consider a numerical example of fuzzy transportation problem which is given below:

	FD1	FD2	FD3	FD4	Supply
FS1	[1, 2, 3, 4]	[1, 3, 4, 6]	[9, 11, 12, 14]	[5, 7, 8, 11]	[1, 6, 7, 12]
FS2	[0, 1, 2, 4]	[-1, 0, 1, 2]	[5, 6, 5, 7, 8]	[0, 1, 2, 3]	[0, 1, 2, 3]
FS3	[3, 5, 6, 8]	[5, 8, 9, 2]	[12, 15, 16, 19]	[7, 9, 10, 12]	[5, 10, 12, 17]
Demand	[5, 7, 8, 10]	[1, 5, 6, 10]	[1, 3, 4, 6]	[1, 2, 3, 4]	

3.2 Two Stage Method

Stage 1:

The fuzzy initial basic feasible solution using Fuzzy Vagol's Approximation Method (FVAM) is

$$[x_{11}^{(1)}, x_{11}^{(2)}, x_{11}^{(3)}, x_{11}^{(4)}] = [-9, 0, 2, 11]$$
$$[x_{12}^{(1)}, x_{12}^{(2)}, x_{12}^{(3)}, x_{12}^{(4)}] = [1, 5, 6, 10]$$
$$[x_{24}^{(1)}, x_{24}^{(2)}, x_{24}^{(3)}, x_{24}^{(4)}] = [0, 1, 2, 3]$$
$$[x_{31}^{(1)}, x_{31}^{(2)}, x_{31}^{(3)}, x_{31}^{(4)}] = [-6, 5, 8, 19]$$
$$[x_{33}^{(1)}, x_{33}^{(2)}, x_{33}^{(3)}, x_{33}^{(4)}] = [1, 3, 4, 6]$$
$$[x_{34}^{(1)}, x_{34}^{(2)}, x_{34}^{(3)}, x_{34}^{(4)}] = [-2, 0, 2, 4]$$

The initial fuzzy transportation cost is $Z = [-55.5, 93, 154, 302.5]$ and the crisp value or rank is $(-55.5 + 93 + 154 + 302.5)/4 = 13.5$.

Stage 2:

The fuzzy optimal solution using FMDM in terms of trapezoidal fuzzy numbers is as follows:

$$x_{12} = [x_{12}{}^{(1)}, x_{12}{}^{(2)}, x_{12}{}^{(3)}, x_{12}{}^{(4)}] = [1, 5, 6, 10]$$
$$x_{13} = [x_{13}{}^{(1)}, x_{13}{}^{(2)}, x_{13}{}^{(3)}, x_{13}{}^{(4)}] = [-9, 0, 2, 11]$$
$$x_{23} = [x_{23}{}^{(1)}, x_{23}{}^{(2)}, x_{23}{}^{(3)}, x_{23}{}^{(4)}] = [0, 1, 2, 3]$$
$$x_{31} = [x_{31}{}^{(1)}, x_{31}{}^{(2)}, x_{31}{}^{(3)}, x_{31}{}^{(4)}] = [-15, 5, 10, 30]$$
$$x_{33} = [x_{33}{}^{(1)}, x_{33}{}^{(2)}, x_{33}{}^{(3)}, x_{33}{}^{(4)}] = [-13, -1, 3, 15]$$
$$x_{34} = [x_{34}{}^{(1)}, x_{34}{}^{(2)}, x_{34}{}^{(3)}, x_{34}{}^{(4)}] = [-2, 1, 4,74]$$

The total fuzzy transportation cost is $Z = [-403, 45.5, 196.5, 645]$ and the crisp value or rank is $(-403 + 45.5 + 196.5 + 645)/4 = 121$.

3.3 Fuzzy Zero Point Method

The optimal solution by using FZPM proposed by Dinegar and Palanivel [12]

$$x_{12} = [x_{12}{}^{(1)}, x_{12}{}^{(2)}, x_{12}{}^{(3)}, x_{12}{}^{(4)}] = [1, 5, 6, 10]$$
$$x_{13} = [x_{13}{}^{(1)}, x_{13}{}^{(2)}, x_{13}{}^{(3)}, x_{13}{}^{(4)}] = [-9, 0, 2, 11]$$
$$x_{23} = [x_{23}{}^{(1)}, x_{23}{}^{(2)}, x_{23}{}^{(3)}, x_{23}{}^{(4)}] = [0, 1, 2, 3]$$
$$x_{31} = [x_{31}{}^{(1)}, x_{31}{}^{(2)}, x_{31}{}^{(3)}, x_{31}{}^{(4)}] = [5, 7, 8, 10]$$
$$x_{33} = [x_{33}{}^{(1)}, x_{33}{}^{(2)}, x_{33}{}^{(3)}, x_{33}{}^{(4)}] = [-9, -1, 3, 11]$$
$$x_{34} = [x_{34}{}^{(1)}, x_{34}{}^{(2)}, x_{34}{}^{(3)}, x_{34}{}^{(4)}] = [1, 2, 3,4]$$

The total fuzzy transportation cost is $Z = [-202.5, 66, 176, 444.5]$ and the crisp value or rank is $(-202.5 + \ 66 + 176 + 444.5)/4 = 121$

Although, the values of the variables as obtained by Pandian and Natarajan [28] are same as what obtain by the authors, but the optimal fuzzy transportation cost by the present method is 121 whereas, as obtained by Panadian method the solution obtained is 132.17.

4 Conclusion

In this paper, the transportation costs are taken as fuzzy numbers which are more realistic and general in nature. Mathematical formulation of fuzzy transportation problem is discussed with relevant numerical example. Numerical example shows that using this formulation one can have the optimal solution as well as the crisp and fuzzy optimal total cost. Moreover, the fuzzy transportation problem of trapezoidal numbers has been transformed into crisp transportation problem using Robust's ranking indices and then obtained the optimal solution FVAM and FZPM. By this formulation technique one can also be used for solving various other problems also like, project schedules, assignment problems, network flow problems etc.

Reference

[1] Ammar E. E., Youness E., "A Study on multi-objective transportation problem with fuzzy numbers", *Applied Mathematics and Computation,* Vol. 6, No. 2, pp. 241-253, 2005.

[2] Ahlatcioglu M., Sivri M., Güzel N., "Transportation of the Fuzzy Amounts Using the Fuzzy Cost", *Journal of Marmara for Pure and Applied Sciences,* Istanbul, Turkey, Vol. 8, 2002.

[3] Abbasbabdy S. and Asad B., "Ranking of fuzzy numbers by sign distance", *Information Science,* Vol. 176, pp. 2405-2416, 2006.

[4] Bellman R. E., Zadeh L. A., "Decision making in a fuzzy environment", *Management Science,* Vol. 17, pp. 141-164, 1970.

[5] Chanas S., Kulej M., "A fuzzy linear programming problem with equality constrains", *Control and Cybernetics,* Vol. 13, pp. 195-201, 1984.

[6] Chanas S., Kolodziejczyk W., Machaj A, "A fuzzy Approach to the Transportation Problem", *Fuzzy Sets and System,* Vol. 13, pp. 211-221.

[7] Chanas S., Kuchat D., "Fuzzy Integer Transportation Problem", *Fuzzy Sets and Systems,* Vol. 98, pp. 291-298, 1998.

[8] Cheng C. H., "A new approach for ranking fuzzy numbers by distance method", *Fuzzy Sets and Systems,* Vol. 95, pp. 307-317, 1998.

[9] Dantzig G. B., "Application of the simplex method to a transportation problem", Chap 23 in Koopmans, Activity Analysis of production and allocation, cowls commission monograph 13, John Wiley and Sons, Inc., New Youk, 1951.

[10] Edward Samuel A, Improved Zero Point Method(IZPM) for the Transportation Problems, Applied Mathematical Sciences, 6, 5421 - 5426, 2012.

[11] Flood, Merrill M. "On the Hitchcock distribution problem", *Pacific Journal of Mathematics* Vol. 3, No. 2, pp. 369—386, 1953. http://projecteuclid.org/euclid.pjm/1103051397.

[12] Fortemps, R. M., "Ranking and defuzzification methods based on are compensation", *Fuzzy Sets and Sysems,* Vol. 82, pp. 319-330, 1996.

[13] Greig, D. M., "Optimization", *Lonman Group Limited,* London, pp. 100-111, 1980.

[14] Geetha S., Nair K. P., "A Stochastic Bottleneck Transportation Problem", *Journal of Operations Research,* Vol. 45, No. 5, pp. 583-588, 1994.

[15] 18Gen M., Ida K. Li Y., "Solving Bicriteria Solid Transportation Problem", *Humans Information and Technology. IEEE International Conference,* Vol. 2, pp. 1200-1207, 1994.

[16] Hitchcock, F., "The Distribution of a Product from Several Sources to Numerous Localities" *Journal of Mathematics and Physics*, Vol. 20, pp. 224-230, 1941.

[17] Kwak, N. K., "Mathematical Programming with Business Applications", *McGraw-Hill Book Company,* New York, pp. 88-149, 1973.

[18] Kikuchi S., "A method to defuzzify the fuzzy number: Transportation problem application", *Fuzzy Sets and Systems,* Vol. 116, pp. 3-9, 2000.

[19] Lai Y. J., Hwang C. L., "Fuzzy Mathematical Programming Methods and Application", *Springer,* Berlin, 1992.

[20] Liu S. T., Kao C., "Solving Fuzzy Transportation Problems based on extension Principle", *European Journal of Operational Research,* Vol. 153, pp. 661-674, 2004.

[21] Mohanaselvi S., Ganesan K., "Fuzzy Optimal Solution to Fuzzy Transportation Problem: A New Approach", *International Journal of Computer Science and Engineering,* Vol. 4, No. 3, pp. 367-375, 2012.

[22] OhEigeartaigh M., "A Fuzzy Transportation Algorithm", *Fuzzy Sets and Systems,* Vol. 8, pp. 235-243, 1982.

[23] Pandian P. and Natarajan G., "A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem", *Applied Mathematical Science,* Vol. 4, pp. 79-90, 2010.

[24] Stephen K.palanivel, "On Trapezoidal Membership Functions in Solving Transportation Problem under Fuzzy Environment", *International Journal of Computational Physical Sciences,* Vol. 1, No. 1, pp. 1-12, 2009.

[25] Shanmugasudari M. and Ganesan K., "A Novel Approach for the Fuzzy optimal solution of Fuzzy Transportation", *International Journal of Engineering Research and Applications,* Vol. 3, No. 1, pp. 1416-1424, 2013.

[26] Wahed W. F. A. E., "A multi-objective transportation problem under fuzziness", *Fuzzy Sets and Systems,* Vol. 117, pp. 27-33, 2001.

[27] Wahed W. F. A. E., Lee S. L., "Interactive fuzzy goal programming for multi objective transportation problems", *Omega,* Vol. 34, pp. 158-166, 2006.

[28] Yager R. R., "A procedure for ordering fuzzy subsets of the unit interval", *Information Sciences,* Vol. 24, pp. 143-161, 1981.

[29] Zadeh L. A., "Fuzzy sets", *Information and Control*, 8, pp. 338–353, 1965.